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Similarity solutions for laminar forced convection heat transfer from wedges to fluids of any Prandtl number

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Abstract—This paper proposes a similarity solution method that provides very accurate solutions for laminar forced convection heat transfer from either an isothermal surface or a uniform-flux boundary to fluids of any Prandtl number. In this work, we demonstrate the similarity solution method with isothermal and uniform-flux wedges as illustrative examples. This method is based on the introduction of a parameter $\lambda = (Pr\,Re)^{1/2}/(1+Pr)^{1/6}$ to properly define the similarity variables. The resulting similarity equations are simple in form and can be readily integrated using the Runge–Kutta scheme to give solutions that are almost identical to the reported exact solutions over the range of vanishingly small Prandtl numbers to infinity. A simple correlation equation for any wedge and any Prandtl number is also presented. This solution method is particularly valuable for the cases where exact solutions are not available.

1. INTRODUCTION

LAMINAR forced convection heat transfer of incompressible Falkner–Skan flows from an isothermal wedge has been studied very extensively [1–22]. Exact solutions of the transformed similarity energy equation were obtained by Pohlhausen [1] and Eckert [2] for fluids which have Prandtl numbers in the range $0.1 \leqslant Pr \leqslant 15$. The exact integral equation can be calculated approximately by a series expansion of the stream function or of the streamwise velocity component. Lighthill [3] approximated the velocity profile in the thermal boundary layer as a linear function of the transverse distance. Lighthill's analysis is asymptotically exact for the case of extremely large Prandtl number. To obtain an accurate solution for finite values of Prandtl number, Spalding [4] extended Lighthill's approximation by taking account of the quadratic term which describes the curved profile. Further extensions were developed [5–8] by using a more precise stream function in the thermal boundary layer to provide accurate solutions for fluids in a wider range of Prandtl numbers. An improved Lighthill's analysis has also been given by Chao [9]. Asymptotic expansions of heat transfer rate for very large [10–12] and very small [10, 13–20] Prandtl numbers have also been reported.

In spite of the extensive studies into this problem, there is still a need for a simple solution method that will give very accurate solutions for fluids of any Prandtl number, especially for the uniform-flux cases. The object of this work is to introduce a similarity solution method that provides exact solutions over the entire range of Prandtl number from vanishingly small to infinity for wedges of various configurations with isothermal or uniform-flux surfaces. This solution method can be applied as well to other forced convection heat transfer problems.

We propose a parameter $\lambda = (Pr\,Re)^{1/2}/(1+Pr)^n$, where $n = 1/4$ for the special case of separated wedge flow (wedge factor $\beta = -0.198838$) and $n = 1/6$ for other cases, to properly define the similarity variable, the reduced stream function, and the dimensionless temperature for the uniform-flux case. These dimensionless variables lead to a single set of similarity boundary-layer equations which can be integrated readily by employing a Runge–Kutta scheme for any β and Pr. The resulting solutions over the range of $Pr = 10^{-4}$ to ∞ are almost identical to the exact solutions. Moreover, Nusselt number is proportional to $Pr^{1/3}$ and $Pr^{1/2}$ for very large and small Prandtl numbers, respectively. For the special case of separated wedge flow, the present results also indicate that Nu is proportional to $Pr^{1/4}$ as $Pr \to \infty$ [6].

2. ANALYSIS

The boundary-layer equations for laminar, incompressible fluid flow over a wedge of angle $\pi\beta$, with negligible dissipation and body force, are well known as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_\infty \frac{\mathrm{d}u_\infty}{\mathrm{d}x} + \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

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NOMENCLATURE

C	constant
C_f	local friction coefficient
C_0	asymptotic value of Nu/λ for $Pr \to 0$
C_∞	asymptotic value of Nu/λ for $Pr \to \infty$
f	reduced stream function, $\Psi/(\alpha\lambda)$
h	local heat transfer coefficient
k	thermal conductivity
m	$\beta/(2-\beta)$
n	exponent
Nu	local Nusselt number, hx/k
Pr	Prandtl number, ν/α
q_w	wall heat flux
Re	local Reynolds number, $u_\infty x/\nu$
T	temperature
T_w	wall temperature
T_∞	temperature of ambient fluid
u	velocity component in the x-direction
u_∞	potential flow velocity
v	velocity component in the y-direction
x	coordinate along the wedge surface
y	coordinate normal to the wedge surface.

Greek symbols

α	thermal diffusivity
β	angle factor of the wedge
η	similarity variable, $(y/x)\lambda$
θ	dimensionless temperature, $(T-T_\infty)/(T_w-T_\infty)$ for isothermal wedge, and $(T-T_\infty)/(q_w x/k)$ for uniform-flux wedge
λ	$\sigma Re^{1/2}$
μ	viscosity
ν	kinematic viscosity
ρ	density
σ	$Pr^{1/2}/(1+Pr)^n$, $n = 1/4$ for $\beta = -0.198838$ and $n = 1/6$ for other β
τ_w	wall shear stress, $\mu(\partial u/\partial y)_{y=0}$
Ψ	stream function.

The boundary conditions are

$$u(x,0) = 0, \quad v(x,0) = 0 \tag{4}$$

$$u(x,\infty) = u_\infty(x) \tag{5}$$

$$T(x,0) = T_w \quad \text{for an isothermal wedge} \tag{6a}$$

$$-k\left(\frac{\partial T}{\partial y}\right)_{y=0} = q_w \quad \text{for a uniform-flux wedge} \tag{6b}$$

$$T(x,\infty) = T_\infty \tag{7}$$

where

$$u_\infty(x) = Cx^m, \quad \text{with } m = \beta/(2-\beta) \tag{8}$$

is the velocity of the potential flow outside the boundary layer.

We propose a parameter

$$\lambda = \sigma Re^{1/2} = (Pr\,Re)^{1/2}/(1+Pr)^n \tag{9}$$

where $n = 1/4$ for the special case of separated wedge flow ($\beta = -0.198838$), otherwise $n = 1/6$, to properly scale the variables for the similarity transformation of the boundary-layer equations (1)–(7). In terms of λ, the similarity variable, the reduced stream function, and the dimensionless temperature are defined, respectively, as

$$\eta = (y/x)\lambda \tag{10}$$

$$f(\eta) = \Psi(x,y)/(\alpha\lambda) \tag{11}$$

and

$$\theta(\eta) = (T-T_\infty)/(T_w-T_\infty) \tag{12a}$$

for an isothermal wedge, or

$$\theta(\eta) = \lambda(T-T_\infty)/(q_w x/k) \tag{12b}$$

for a uniform-flux wedge. The parameter and the variables are so defined that all the following appropriate scales of the longitudinal velocity and the local Nusselt number for extremely large and vanishingly small Prandtl numbers [21] are satisfied:

$$u \sim Pr^{-1/3} u_\infty \tag{13a}$$

$$Nu \sim Pr^{1/3} Re^{1/2} \quad \text{for } Pr \gg 1 \tag{14a}$$

and

$$u \sim u_\infty \tag{15}$$

$$Nu \sim Pr^{1/2} Re^{1/2} \quad \text{for } Pr \ll 1. \tag{16}$$

For the special case of separated wedge flow ($\beta = -0.198838$), the scaling laws (13a) and (14a) are replaced by

$$u \sim Pr^{-1/2} u_\infty \tag{13b}$$

$$Nu \sim Pr^{1/4} Re^{1/2} \quad \text{for } Pr \gg 1. \tag{14b}$$

The utilizing of the similarity variable and reduced stream function leads to the similarity transformation of the momentum equation (2) as

$$Prf''' + \frac{m+1}{2} ff'' + m[(1+Pr)^{4n} - f'f'] = 0 \tag{17}$$

with boundary conditions

$$f(0) = 0, \quad f'(0) = 0 \tag{18}$$

$$f'(\infty) = (1 + Pr)^{2n}. \tag{19}$$

In addition, the velocity components are obtained as

$$u/u_\infty = (1 + Pr)^{-2n} f'(\eta) \tag{20}$$

and

$$v = -(\alpha/x)\lambda\left[\frac{m+1}{2}f + \frac{m-1}{2}\eta f'\right]. \tag{21}$$

It can be seen from equation (20) that the scaling laws (13) and (15) for the longitudinal velocity are satisfied respectively for very large and very small Prandtl numbers.

Since the Prandtl number appears explicitly in equations (17) and (19), solutions of $f''(0)$ are expected to be a function of Prandtl number. However, the combination of the local friction coefficient $C_f = 2\tau_w/(\rho u_\infty^2)$ and the square root of the local Reynolds number $Re = u_\infty x/\nu$, namely

$$C_f Re^{1/2} = 2(1 + Pr^{-1})^{-1/2} f''(0) \tag{22}$$

is independent of the Prandtl number. For example, the computed values of $f''(0)$ for $\beta = 0$, from the numerical integration of equations (17) subject to boundary conditions (18) and (19) by using the Runge–Kutta scheme, vary from 33.20729 for $Pr = 0.0001$ to 0.332090 for $Pr = 10{,}000$, while the derived values of $C_f Re^{1/2}$ are in the range of 0.66410–0.66414.

The energy equation (3) with boundary conditions (6) and (7) can be reduced to

$$\theta'' + \frac{m+1}{2}f\theta' = 0 \tag{23}$$

$$\theta(0) = 1, \quad \theta(\infty) = 0 \tag{24a, b}$$

for isothermal wedges; and

$$\theta'' + \frac{m+1}{2}f\theta' + \frac{m-1}{2}f'\theta = 0 \tag{25}$$

$$\theta'(0) = -1, \quad \theta(\infty) = 0 \tag{26a, b}$$

for uniform-flux wedges. Although the Prandtl number does not appear explicitly in the reduced energy equation (23) or (25) and boundary conditions (24) or (26), the heat transfer results still depend on the Prandtl number since f and f' are functions of the Prandtl number.

Equations (23) and (24) for the isothermal case or equations (25) and (26) for the uniform-flux case are integrated numerically in conjunction with the reduced momentum equation (17) and boundary conditions (18) as well as another boundary condition $f''(0) = 0.5(1 + Pr^{-1})^{1/2}(C_f Re^{1/2})$ from equation (22). The values of $C_f Re^{1/2}$ are known as 0, 0.66412, 1.51490 and 2.465175 for $\beta = -0.198838$, 0, 0.5 and 1, respectively. A fourth-order Runge–Kutta scheme with variable integration step size is employed for the numerical solution. A convergence criterion of 10^{-6} at the edge of the boundary layer was used in the computations.

Table 1. Similarity solutions of $Nu/\lambda = -\theta'(0)$ for isothermal wedges

Pr	$\beta = -0.198838$	$\beta = 0$	$\beta = 0.5$	$\beta = 1$
0 [22]	0.538076	0.564190	0.651470	0.797885
0.0001	0.528167	0.558777	0.647376	0.793809
0.001	0.508230	0.547663	0.638768	0.785306
0.01	0.455989	0.516758	0.614372	0.760987
0.1	0.356835	0.449911	0.559225	0.705249
1	0.261293	0.372722	0.493968	0.640326
10	0.229354	0.343388	0.477039	0.631365
100	0.224976	0.339208	0.482208	0.644454
1000	0.224520	0.338766	0.486599	0.653023
10000	0.224474	0.338722	0.488816	0.657181
∞	0.224469	0.338720	0.490753	0.660766
∞ [23]	0.224	0.339	—	0.661

Table 2. Similarity solutions of $\theta(0)$ for uniform-flux wedges

Pr	$\beta = -0.198838$	$\beta = 0$	$\beta = 0.5$	$\beta = 1$
0 [22]	—	1.12838	—	1.25331
0.0001	1.15129	1.14545	1.18050	1.25953
0.001	1.22509	1.18143	1.20127	1.27339
0.01	1.44399	1.28721	1.26263	1.31408
0.1	2.00751	1.55114	1.41813	1.41794
0.3	2.42764	1.74043	1.52993	1.49170
0.7	2.76231	1.88681	1.61402	1.54446
1	2.88864	1.94108	1.64379	1.56170
3	3.17705	2.06301	1.70336	1.58888
7	3.29585	2.11245	1.71961	1.58763
10	3.32611	2.12497	1.72148	1.58387
100	3.39488	2.15333	1.71001	1.55170
1000	3.40219	2.15633	1.69734	1.53134
10000	3.40292	2.15663	1.69084	1.52165
∞	3.40300	2.15667	1.68515	1.51340
∞ [22]	—	—	—	1.513

3. RESULTS AND DISCUSSION

Similarity solutions of $-\theta'(0)$ for the case of isothermal wedges and those of $\theta(0)$ for uniform-flux wedges over the range of $Pr = 10^{-4}$ to ∞ are listed in Tables 1 and 2, respectively. Available closed form solutions for $Pr \to 0$ and $Pr \to \infty$ [22, 23] are also presented in these tables for comparison. The heat transfer rate can be expressed in terms of local Nusselt number $Nu = hx/k$ which is related to $\theta'(0)$ and $\theta(0)$ by

$$Nu/\lambda = -\theta'(0) \tag{27}$$

for the isothermal wedges; and

$$Nu/\lambda = 1/\theta(0) \tag{28}$$

for the uniform-flux wedges. Variations of Nu/λ with Prandtl number over the range from 10^{-4} to 10^4 are illustrated in Figs. 1 and 2, respectively, for isothermal and uniform-flux cases. Asymptotic values of the

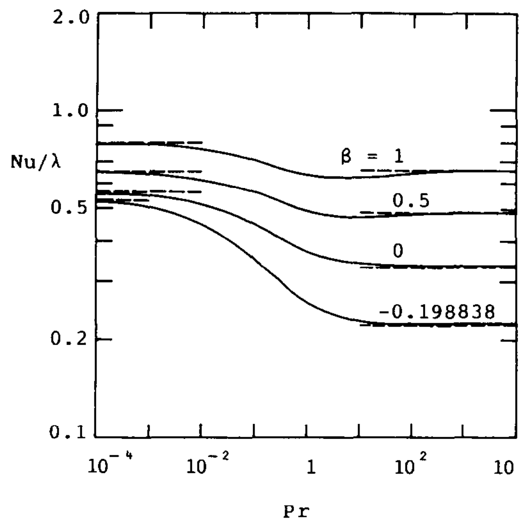

FIG. 1. Variations of heat transfer rates with Prandtl number for isothermal wedges.

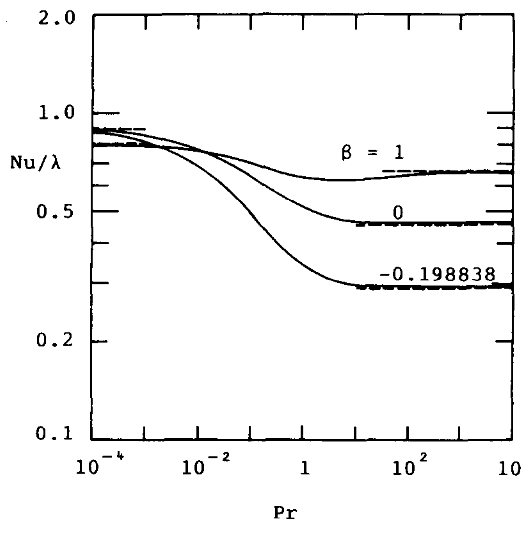

FIG. 2. Variations of heat transfer rates with Prandtl number for uniform-flux wedges.

closed form solutions [22, 23] for $Pr \to 0$ or $Pr \to \infty$, and present similarity solutions for $Pr \to \infty$ are also presented. As shown in these figures, the value of Nu/λ for $\beta \leqslant 0$ decreases gradually from a constant C_0 for the limiting case of vanishingly small Pr to another constant C_∞ for the other limit of infinite Pr. Moreover, Nu/λ becomes nearly constant and is very close to the asymptotes for small and large Prandtl numbers. Consequently, equation (27) or (28) can be reduced to $Nu(Pr\,Re)^{-1/2} = C_0$ and $Nu(Pr^{-1/3} \times Re^{-1/2}) = C_\infty$, respectively, for the limiting cases of $Pr \to 0$ and $Pr \to \infty$. For the special case of separated wedge flow ($\beta = -0.198838$), $Nu/\lambda = Nu(Pr^{-1/4} \times Re^{-1/2}) = C_\infty$ for $Pr \to \infty$. It becomes quite clear that the scaling laws (14) and (16) for the limiting cases are satisfied with the expression of Nu/λ.

It is worth noting that $\lambda = (Pr\,Re)^{1/2}/(1 + Pr)^{1/6}$ is not the unique form which leads to Nu/λ and u that satisfy the scaling laws (13)–(16) for the limiting cases. Indeed, these scaling laws are satisfied by using a general form of $\lambda = (Pr\,Re)^{1/2}/(1 + Pr^a)^b$ or, equivalently, $\lambda = Pr^{1/3}Re^{1/2}/(1 + Pr^{-a})^b$, with $ab = 1/6$.

For a closer comparison, the present similarity solutions of $Nu\,Re^{-1/2} = -\sigma\theta'(0)$ and exact solutions of $Nu\,Re^{-1/2} = (b_0'/B)/(2 - \beta)^{1/2}$ [6, 8] over the range of $Pr = 10^{-4}$–10^4 are listed in Table 3 for the isothermal wedges of $\beta = -0.198838$, 0, 0.5 and 1. The two solutions are identical even to the fifth significant digit for most of the cases.

The similarity solutions of $Nu\,Re^{-1/2} = \sigma/\theta(0)$ for uniform-flux wedges are compared in Table 4 with $Nu\,Re^{-1/2} = 1/[(2 - \beta)^{1/2}\,\theta_s(0)]$ of Chao and Cheema [22] over the range of $Pr = 0.01$–100. It is seen that these solutions are in excellent agreement. In addition, for the case of forward stagnation ($\beta = 1$) the similarity solution of $Nu/\lambda = 1/1.51340$ (Table 2) for $Pr \to \infty$ is in excellent agreement with the closed form solution of $Nu\,Re^{-1/2}\,Pr^{-1/3} = 1/1.513$ [22].

It is very interesting to note that the reduced energy equations (23) and (25), for isothermal boundary and uniform-flux surfaces, respectively, are identical for the case of forward stagnation ($\beta = 1$). Although boundary conditions (24a) and (26a) are different, the solutions of $Nu/\lambda = -\sigma\theta'(0)$ for the isothermal surface and $Nu/\lambda = \sigma/\theta(0)$ for the uniform-flux boundary are almost the same.

A simple correlation equation for forced convection heat transfer rates from wedges to fluids of any Prandtl number from 0 to infinity is proposed here as

$$Nu/(Re\,Pr)^{1/2} = A/(B + Pr)^n \qquad (29)$$

with $A = C_\infty$ and $B = (C_\infty/C_0)^{1/n}$, where C_∞ is the asymptotic value of Nu/λ as $Pr \to \infty$; and C_0 is that of $Nu/\lambda = Nu/(Re\,Pr)^{1/2}$ as $Pr \to 0$. The values of C_0 and C_∞ for various β are listed in Tables 1 and 2. Some of them can also be calculated from the available asymptotic equations [22, 23]. As has been mentioned, the exponent $n = 1/4$ for the special case of separated wedge flow and $n = 1/6$ for other cases. The correlation equation (29) is applicable to wedges of various configurations imposed with either an isothermal or a uniform-flux boundary. When compared with the exact solutions, the maximum error of this correlation equation is less than 10.6% for all the cases listed in Tables 1 and 2 over the range of $Pr = 0$–∞. The maximum discrepancy between equation (29) and the exact solutions occurs at the Prandtl number that is of the same order as B. A more precise correlation equation is presently being studied.

Representative dimensionless temperature distributions are presented in Figs. 3 and 4, respectively, for the cases of isothermal plate and uniform-flux plate and for $Pr = 0.001$, 0.01, 0.1, 1 and 1000. The corresponding temperature distributions for the case of separated wedge flow are also demonstrated in Figs. 5 and 6. These figures show that the dimensionless

Table 3. Comparison of the present results of $Nu\,Re^{-1/2} = -\sigma\theta'(0)$ with exact solutions of $Nu\,Re^{-1/2} = (b_0'/B)/(2-\beta)^{1/2}$ in refs. [6, 8] for isothermal wedges

	$\beta = -0.198838$		$\beta = 0$		$\beta = 1$	
Pr	Present	Exact	Present	Exact	Present	Exact
0.0001	0.528154(−2)	0.528150(−2)	0.558768(−2)	0.558773(−2)	0.793796(−2)	0.793791(−2)
0.001	0.160676(−2)	0.160675(−1)	0.173157(−1)	0.173156(−1)	0.248294(−1)	0.248290(−1)
0.01	0.454856(−1)	0.454853(−1)	0.515902(−1)	0.515884(−1)	0.759726(−1)	0.759720(−1)
0.1	0.110184	0.110184	0.140032	0.140029	0.219505	0.219503
1	0.219720	0.219720	0.332058	0.332057	0.570466	0.570466
10	0.398252	0.398250	0.728148	0.728136	1.33880	1.33880
100	0.709669	0.709668	1.57186	1.57183	2.98634	2.98633
1000	1.26225	1.26225	3.38710	3.38707	6.52914	6.52914
10000	2.24468	2.2446	7.29742	7.29734	14.1583	14.158

Table 4. Comparison of present results of $Nu\,Re^{-1/2} = \sigma/\theta(0)$ with $Nu\,Re^{-1/2} = 1/[(2-\beta)^{1/2}\theta_s(0)]$ of Chao and Cheema [22] for uniform-flux wedges

	$\beta = 0$		$\beta = 0.5$		$\beta = 1$	
Pr	Present	ref. [22]	Present	ref. [22]	Present	ref. [22]
0.01	0.775587(−1)	0.775584(−1)	0.790684(−1)	0.790673(−1)	0.759728(−1)	0.759723(−1)
0.1	0.200655	0.200654	0.219476	0.219475	0.219505	0.219501
1	0.458971	0.458970	0.541978	0.541979	0.570467	0.570467
10	0.997888	0.997879	1.23178	1.23180	1.33880	1.33887
100	2.15196	2.15194	2.70988	2.71018	2.98634	2.98784

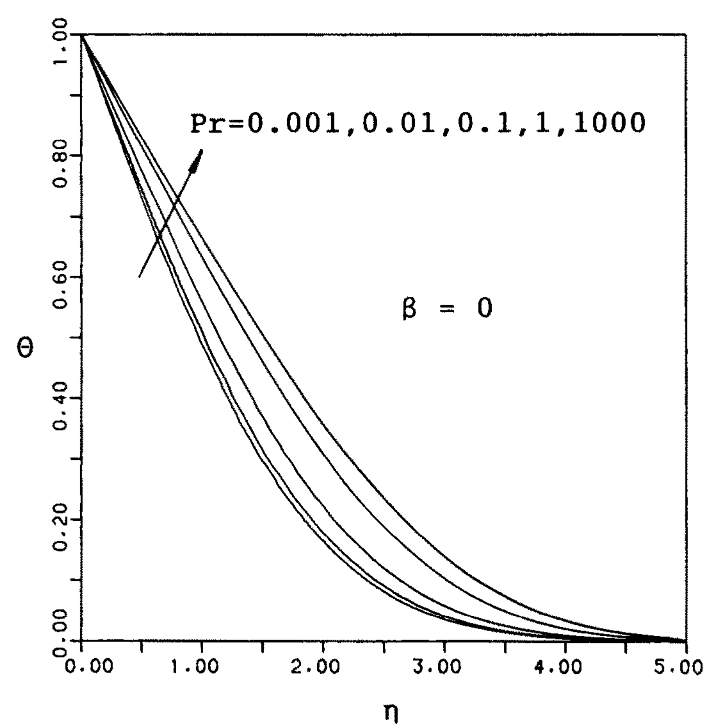

FIG. 3. Dimensionless temperature profiles for an isothermal flat plate.

temperature profiles are similar for all fluids, especially for $Pr > 1$.

4. CONCLUSIONS

A similarity solution method was proposed in this paper for the forced convection heat transfer from isothermal or uniform-flux surfaces to fluids of any Prandtl number from vanishingly small values to infinity. As illustrated by the cases of wedges of various configurations, the similarity equations resulting from suitably defined similarity variables are simple in form and can be integrated readily to give solutions that are identical to the reported exact solutions for isothermal wedges. This solution method is particularly valuable for the cases of which the exact solutions are not available.

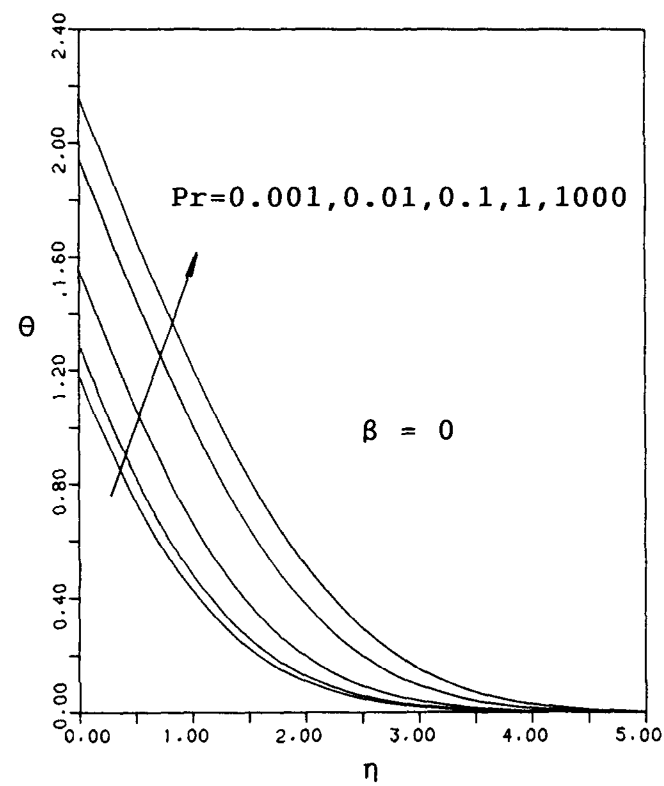

FIG. 4. Dimensionless temperature profiles for a uniform-flux plate.

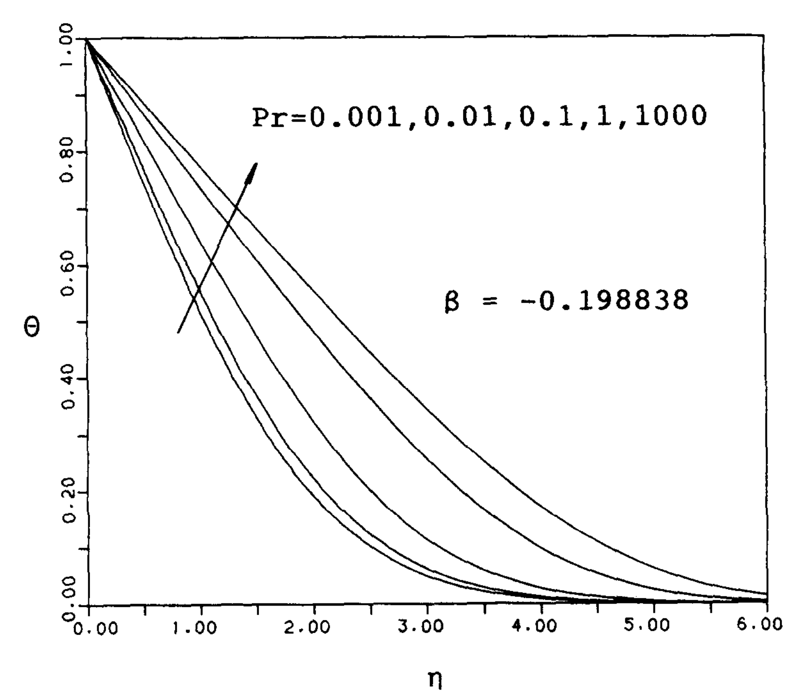

FIG. 5. Dimensionless temperature profiles for separated wedge flow, isothermal case.

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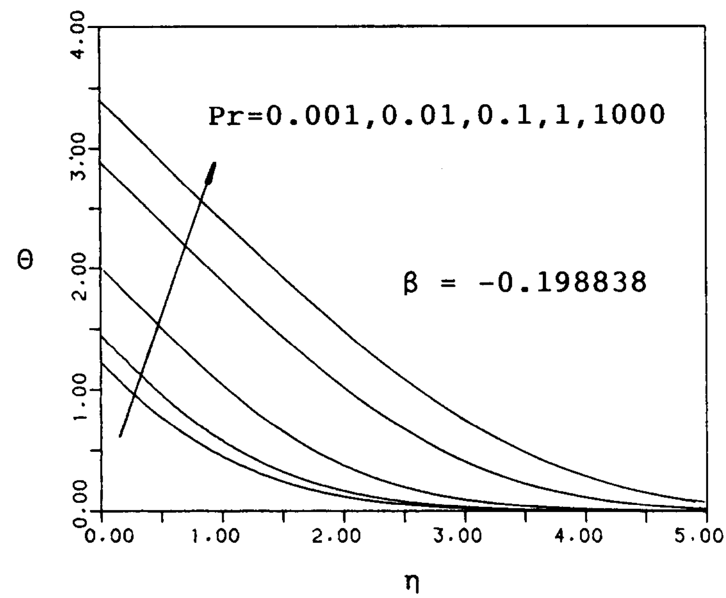

FIG. 6. Dimensionless temp

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SOLUTIONS DE SIMILITUDE DE LA CONVECTION THERMIQUE LAMINAIRE, FORCEE SUR DES DIEDRES POUR DES FLUIDES A NOMBRE DE PRANDTL QUELCONQUE

Résumé—On propose une méthode de résolution par similitude qui fournit des solutions très précises pour la convection thermique, laminaire, forcée sur des surfaces soit isothermes soit à flux uniforme, avec des fluides à nombre de Prandtl quelconque. On traite avec cette méthode le cas des dièdres isothermes ou à flux uniforme, La méthode est basée sur l'introduction d'un paramètre $\lambda = (Pr\,Re)^{1/2}/(1+Pr)^{1/6}$ pour définir convenablement les variables de similitude. Les équations qui en résultent sont simples et peuvent être intégrées directement, en utilisant la méthode Runge–Kutta, pour donner les solutions identiques aux solutions exactes connues, dans le domaine de nombre de Prandtl entre les valeurs proche de zéro et l'infini. On présente aussi une formule pour un dièdre et un nombre de Prandtl quelconques. Cette méthode de résolution est particulièrement utile lorsque les solutions exactes ne sont pas disponibles.

ÄHNLICHKEITSLÖSUNGEN FÜR DEN WÄRMEÜBERGANG BEI LAMINARER, ERZWUNGENER KONVEKTION VON KEILEN AN FLUIDE MIT BELIEBIGER PRANDTLZAHL

Zusammenfassung—Dieser Aufsatz schlägt eine Ähnlichkeitsmethode vor, die sehr genaue Lösungen für den Wärmeübergang bei laminarer erzwungener Konvektion von einer isothermen oder gleichförmig beheizten Oberfläche an Fluide mit beliebiger Prandtl-Zahl liefert. In dieser Arbeit wird die Ähnlichkeitsmethode beispielhaft mit beiden Randbedingungen an keilförmigen Körpern demonstriert. Das Verfahren basiert auf der Einführung eines Parameters $\lambda = (Pr\,Re)^{1/2}/(1+Pr)^{1/6}$, um die Ähnlichkeitsvariablen korrekt zu definieren. Die resultierenden Ähnlichkeitsgleichungen haben eine einfache Form und können ohne weiteres mit dem Runge–Kutta-Verfahren integriert werden. Es ergeben sich Lösungen, die fast identisch mit angegebenen exakten Lösungen sind, und zwar für den Bereich von verschwindend kleinen bis zu unendlich großen Prandtl-Zahlen. Eine einfache Korrelationsgleichung für beliebige Keilform und beliebige Prandtl-Zahl wird ebenfalls vorgestellt. Diese Lösungsmethode ist dann besonders wertvoll, wenn keine exakte Lösungen verfügbar sind.

АВТОМОДЕЛЬНЫЕ РЕШЕНИЯ ЗАДАЧИ ДЛЯ ТЕПЛООТДАЧИ ОТ КЛИНА К ЖИДКОСТИ С ПРОИЗВОЛЬНЫМ ЧИСЛОМ ПРАНДТЛЯ ДЛЯ ЛАМИНАРНОЙ ВЫНУЖДЕННОЙ КОНВЕКЦИИ

Аннотация—Предложен метод получения весьма точных автомодельных решений задачи для теплоотдачи либо от изотермической поверхности или от поверхности с однородным потоком тепла к жидкостям с произвольным числом Прандтля при ламинарной вынужденной конвекции. В качестве примера приведен случай изотермического клина и клина с однородным тепловым потоком. Метод основан на введении параметра при надлежащем выборе автомодельных переменных. Полученные уравнения просты по форме и легко интегрируются методом Рунге–Кутта; получены решения, которые почти совпадают с известными точными решениями в диапазоне чисел Прандтля от предельно малых до бесконечности. Представлено простое критериальное уравнение для произвольного угла раствора клина и числа Прандтля. Такой же подход особенно ценен, когда невозможно получить точное решение.